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A new breed of Monte Carlo to meet FRTB computational challenges

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Acknowledgement & Disclaimer

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The opinions expressed in this presentation and on the following slides are solely those of the presenter and not necessarily those of Natixis.

Summary

1. **FRTB main concepts (SA – IMA)**
2. **Monte Carlo optimization techniques**

Summary

1. **FRTB main concepts (SA – IMA)**
2. Monte Carlo optimization techniques

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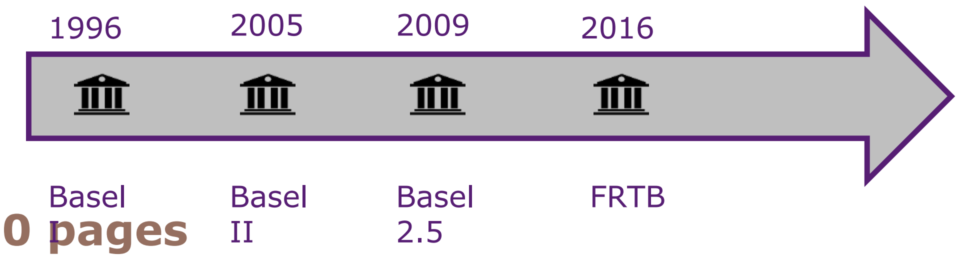
A lookback at previous risk regulations



- **Basel I - 1996 : 20 pages**
  - Regulatory capital requirements for market risk are amended to the overall capital requirements accord
- **Basel II – 2005 : hundred of pages**
  - Changes to existing market risk regime are performed in order to foster international convergence
- **Basel 2.5 - 2009 : thousand of pages**
  - Extensive amendments as consequence of the Global Financial Crisis, focusing on default and migration risk, and treatment of securitizations
- **FRTB - 2016 : 90 pages**
  - Complete overhaul of the existing framework in various areas, like risk measurement methods, placing a large strain on bank’s quantitative finance resources. Banks have to be compliant by December 31 2019

2 FRTB Main Concepts

Standardized Approach (SA)

- For capital requirement calculation
- Fallback or floor for IMA approach
- Consideration of hedging and diversification benefit
- Mandatory calculation monthly
- **Generate higher capital charges than IMA** (TBC)
- Sensitivities based
- Default risk and residual risk add-ons

Internal Model Approach (IMA)

- Switch from VaR to Expected Shortfall (ES)
- Market Risk Illiquidity taken into account
- Default Risk Charge (DRC)
- Strict regulatory led approval process (P&L Attrib, Backtesting)
- Additional charge for Non Modelable Risk factors (NMRF) and default risk

Banking and Trading Books Boundary

- Identification of trading accounts and trader
- Formalization of the business strategy
- Weekly risk reporting requirements for defined desks
- Regulatory led approval process for the proposed desks

FRTB : Eligibility to IMA

Eligibility to IMA approach per desk has to be homologated by the regulator:

Bank Wide Assessment

Desk Level Model Approval

Risk Factors

Homologation

P&L Attribution:
- Monthly computation

Test robustness
- Backtesting

Fallback to SA :
- 4 breaches in 12 months

At desk level:
One have to make sure
- Real transaction prices
- Sufficient observation criteria

Standard Approach (SA)
Risk Capital Charges under Sensitivities method

Linear Risk
(*Delta and Vega*)

Non Linear Risk
(Curvature)

Risk weight sensitivities

$$WS_k = RW_k.\, s_k$$

Where RW_k *is regulator prescribed risk weight*
s_k *is the net sensitivity for risk factor k.*

$$CVR_k = -min\begin{bmatrix}\sum_i V_i(x_k^{RW^{(curvature)}}+) - V_i(x_k) - RW_k^{(curvature)}.\, s_{ik} \\ \sum_i V_i(x_k^{RW^{(curvature)}}-) - V_i(x_k) + RW_k^{(curvature)}.\, s_{ik}\end{bmatrix}$$

Aggregate within bucket b

$$K_b = \sqrt{\sum_k WS_k^2 + \sum_k \sum_{k\neq l} \rho_{kl}.\, WS_k.\, WS_l}$$

Given correlation ρ_{pk}

$$K_b = \sqrt{max(0, \sum_k max(CVR_k, 0)^2 + \sum_k \sum_{k\neq l} \rho_{kl}.\, CVR_K.\, CVR_l.\, \psi(CVR_k, CVR_l))}$$

Aggregate across buckets

$$S_b = max[min[\sum_k WS_k, K_b], -K_b]$$

$$S_c = max[min[\sum_k WS_k, K_c], -K_c]$$

$$S_b = max[min(\sum_k CVR_k, K_b), -K_b]$$

$$S_c = max[min(\sum_k CVR_k, K_c), -K_c]$$

$$Linear\ Risk\ Charge = \sqrt{\sum_b K_b^2 + \sum_b \sum_{c\neq b} \gamma_{bc}.\, S_b.\, S_c}$$

$$Non\ Linear\ Risk\ Charge = \sqrt{max(0, \sum_b K_b^2 + \sum_b \sum_{b\neq c} \gamma_{bc}.\, S_b.\, S_c)}$$

Where γ_{bc} *are correlations,*
b and c are risk buckets

Linear Risk Charge

Standard Approach (SA)
Default Risk Charge challenges

Calculating the DRC consists on several computations:

- **Gross JTD :**
  - For each instrument, and for each equity underlying (Obligor), compute **the impact of the default of the obligor.** This step produces a gross Long / Short JTD per Obligor and per Instrument.
- **Net JTD :**
  - For each Obligor, apply a netting algorithm over all the positions of the bank to obtain a net Long / Short JTD per Obligor.
- **Hedge Benefit :**
  - Application of a partial hedge benefit ratio to account for a partial offset of long and short exposures in distinct Obligors.
- **Risk Weights :**
  - For each of the three reglementary buckets, assign a rating grade to each Obligor and compute the DRC per bucket using the corresponding risk weights.

Internal Model Approach (IMA) General description

The idea is of the IMA is to better take into account :

- Tail risks
- Liquidity risk : Regulator imposed Liquidity Horizon per asset types
- Factor in default risk for select subset of asset classes
- Clearly separate modellable and non modellable risk factors
- Regulatory prescribed list of risk categories

IMA main items:

- A switch to from VaR to Expected Shortfall
- Regulator defined Liquidity horizons to be factored in ES computations
- Default Risk Charge
- Non Modellable Risk Factors

Internal Model Approach (IMA)
A switch to Expected Shortfall

- **Calculated daily**
- **97.5th percentile, one-tailed confidence level is to be used**
- **Quantitative standards**
  - A switch to from VaR to Expected Shortfall
  - Regulator defined Liquidity horizons to be factored in ES computations
  - Separate model to measure default (IMA Default Risk Charge, IMA DRC)
  - Non Modellable Risk Factors

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Internal Model Approach (IMA) A switch to Expected Shortfall

- Stressed expected shortfall computed with all risk factors shocked
- Expected Shortfall addiotionally computed for shocks of each risk factor, all others held constant

$$ES = \sqrt{(ES_T(P)^2) + \sum_{j \geq 2} (ES_T(P, j)\sqrt{\frac{(LH_j - LH_{j-1})}{T}})^2}$$

- T: length of the base horizon (**10 days**)
- $ES_T(P)$ is ES at horizon T of a portfolio P constituted of positions p_i with respect to shocks **to all risk factors** valid for positions within P
- $ES_T(P, j)$ is the ES at horizon T (**10 days**) of a portfolio P (positions p_i) with respect to shocks for **each position p_i** in the subset of risk factors Q(p_i, j) **with all other risk factors held constant**
- ES at horizon T ($ES_T(P)$ and $ES_T(P, j)$ must be calculated for changes in risk factors over the time interval T with full revaluation
- Q(p_i, j) is the subset of risk factors whose liquidity horizons for the desk where p_i is booked are at least as long as LH_j
- The timeseries of changes in risk factors over the base time interval T (**10 days**) may be determined by overlapping intervals
- LH_j Liquidity Horizon j

Internal Model Approach (IMA)
IMCC Calculation

Desk Level Capital Charge

Weighted sum of constrained and unconstrained IMCC :

$$IMCC = \rho(IMCC(C)) + (1-\rho)(\sum_{i=1}^{R} IMCC(C_i))$$

Ratio of ES on reduced and full RF set

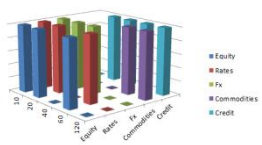
$$IMCC(C) = ES_{R,S} \times \frac{ES_{F,C}}{ES_{R,C}}$$

$$\sum_{i=1}^{R} IMCC(C_i) = \sum_{i=1}^{R} ES_{R,S,i} \times \frac{ES_{F,C,i}}{ES_{R,C,i}}$$

$$= IMCC_{Equity}(C) + IMCC_{Rates}(C) + IMCC_{Fx}(C) + IMCC_{Credit}(C) + IMCC_{Commo}(C)$$

Shock to RF per Liquidity Horizon

$$ES = \sqrt{(ES_T(P)^2) + \sum_{j\geq 2}(ES_T(P,j)\sqrt{\frac{(LH_j - LH_{j-1})}{T}})^2}$$



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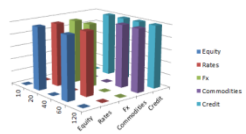

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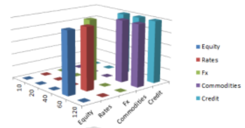

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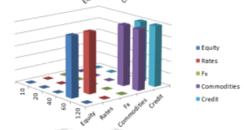

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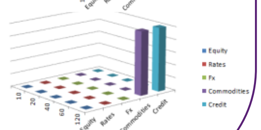
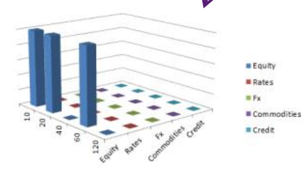


$$ES = \sqrt{(ES_T(P)^2) + \sum_{j\geq 2}(ES_T(P,j)\sqrt{\frac{(LH_j - LH_{j-1})}{T}})^2}$$



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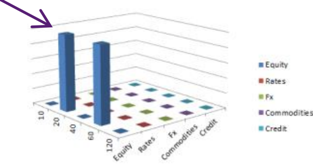

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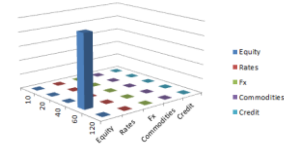

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Summary

1. FRTB main concepts (SA – IMA)
2. **Monte Carlo optimization techniques**

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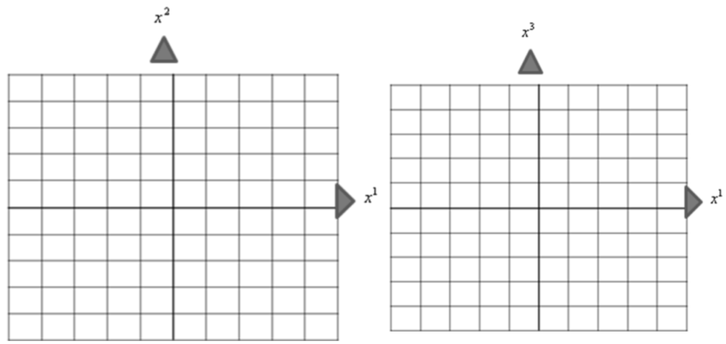
Ideas

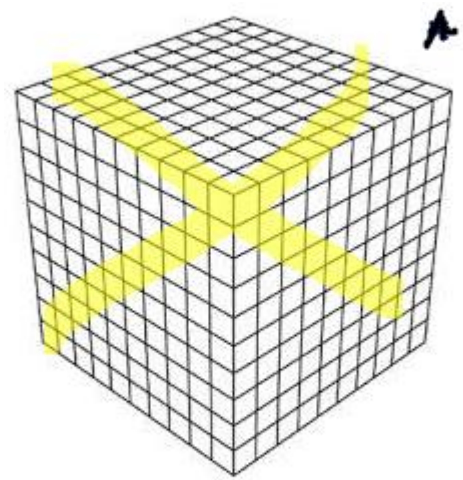
1. **Shadow grid**
2. **Hot Spot Monte Carlo**
3. **Transforming trajectories**

Shadow Grid

Non Parametric approach:

- We put values in a grid and proceed to the algorithm described below:
- We decompose our n dimensional cube in n 2-dimensional projections





Values in a grid with a valuation algorithm as follows:

- We assume that we would like to calculate the function $u(x_1,\ldots,x_n)$
- We locate each couple of coordinates (x_i, x_j) within the grids.
- We use a bi-cubic spline interpolation to value a bi-dimensional function $v_{i,j} = u(x_1(0),..x_i,.x_j, x_n(0))$:here only two variables $(x_i,.x_j)$ have moved.
- We use a cubic spline interpolation to value a one dimensional function $u_i = u(x_1(0),..x_i,.x_j(0), x_n(0))$: here this is a special case of the previous where just one variable has moved.

- We use the reconstruction formula based on Taylor

$$
\begin{aligned}
u(x_1,\ldots,x_n) - u(0) &= \sum_{i=1}^{n}(u_i - u(0)) \\
&+ \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left(v_{i,j} - u(0) - (u_i - u(0)) - (u_j - u(0))\right)
\end{aligned}
$$

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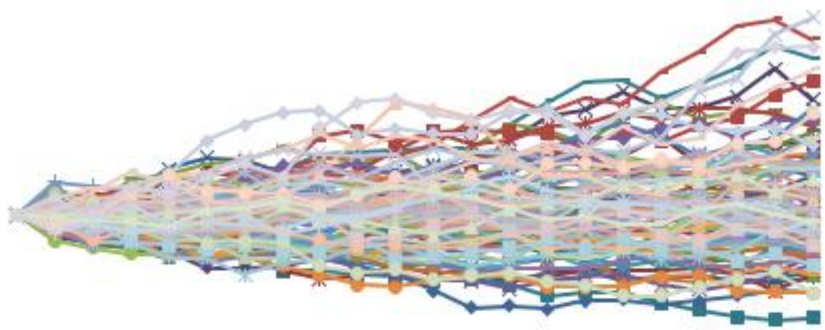
Shadow Grid

- **Efficient FO pricing : Pricing grid capacity estimates**

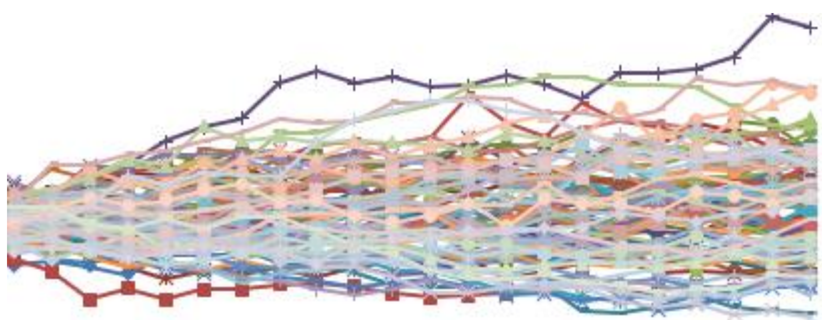
Fast Analytical prices (such as Vanillas, TRS, Swaps)	7 (S) x 3 (vol) + 3(repo) + 3(div) + 3(rates) around 30 pricings necessary.
Slow Analytical prices (such as variance swaps under cash dividend assumption)	Idem.
PDE pricing (barrier options, American options)	7 (S) x 3 (vol) + 3 (skew) + 3(repo) + 3(div) + 3(rates) around 33 pricings.
Monte Carlo Flow Business or Structured business with a usage of aggregators (such as Volatility swaps, Autocall on basket or worst of)	7 (S) x 3 (vol) + 3 (skew) + 3(repo) + 3(div) + 3(rates) around 33 pricings.
Monte Carlo Structured Business (general case)	7 (S) x 3 (vol) + 3 (skew) + 3(repo) + 3(div) + 3(rates) around 33 pricings. Based on previous analysis we have the possibility to use one global pricing which randomizes the payoff and extracts a series of grid prices.
Convertible Bonds	Like a PDE approach

Hot Spot Data Model Diffusion

- Classical simulation for pricing



- Hot Spot simulation for multiple initial condition pricing



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Hot Spot Data Model Diffusion

- Where it comes from?
- Old recipe to stabilise the greeks within a LSM method
- Used in the case of the multi asset Uncertain volatility correlation model

Hot Spot Data Model Diffusion

Efficient FO pricing

Monte Carlo Structured business (general case) : The idea is to randomize the initial conditions, combined with a few scenario

Conceptually:

- If the pricer is $f(x, y)$ where x is the state variable of the diffusion model and y is the state variables of the data model involved in the scenario engine.
- We introduce σ_x, the volatility of the Monte Carlo state variable and σ_y, the volatility of the data model. We assume that the is a correlation ρ_{xy} standing between the two types of variables.

Our task is to calculate the maturity scenarios.

- We extend the existing Monte Carlo by randomizing each path using the following mechanism:

$$\left(x\left(1+\sigma_x \varepsilon_x \sqrt{T}\right), y\left(1+\sigma_x\left(\rho_{xy}\varepsilon_x + \sqrt{1-\rho_{xy}^2}\,\varepsilon_y\right)\sqrt{T}\right)\right)$$

Where $\varepsilon_x, \varepsilon_y$ are two independent normal random variables.

Transforming trajectories

Efficient FO pricing through

1. Architectural building

2. Computational ordering

σ_x

1. DRC challenges

- Calculating the Gross Long / Short JTD per instrument and per Obligor is the first step and the cornerstone of the DRC computation.

- It consists on simulating the default of the underlyings (Obligor per Obligor) then calculating the impact of such defaults.

This specification raises some technical issues:

- What does it mean to simulate the default of an index (e.g. S&P 500), an ETF, a Basket, etc. ?
  *the FRTB guidelines specify that a **look through approach** should be applied.*

- How should we perform the simulations such that the computation time remains bounded ?
  *for an exotic call option on S&P500 priced via 200K Monte Carlo simulations, should we simulate the default of each component of the S&P 500 ? (200K*500 = 100 Millions simulations)*

- What if an Obligor exists within two underlyings of the option?
  for a call option on both CAC 40 and FP Total, should we perform one global or two partial pricings ?

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2. Look Through Approach

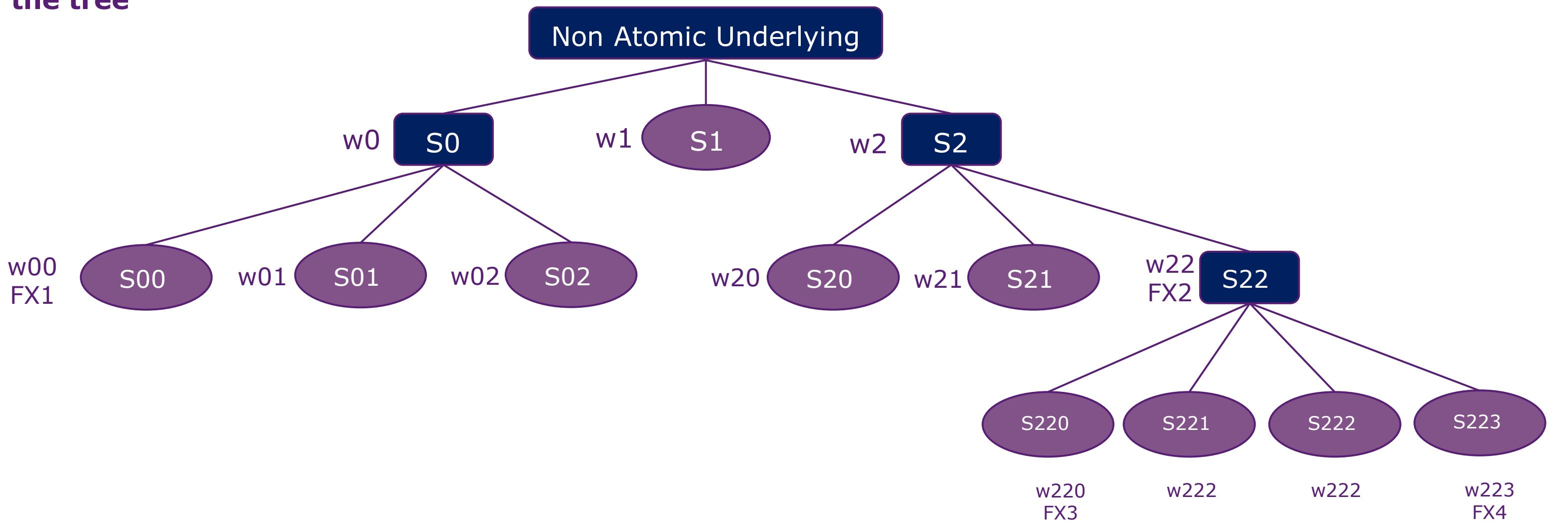
In order to compute the Gross JTD per Obligor for an instrument having a “non atomic” underlying (index / ETF / Fund / Hedge Fund / Basket / etc.), we need to identify two set of parameters :

- **Composition**
  First, we need to identify the list of Obligors on which depends each “non atomic” underlying of the instrument.

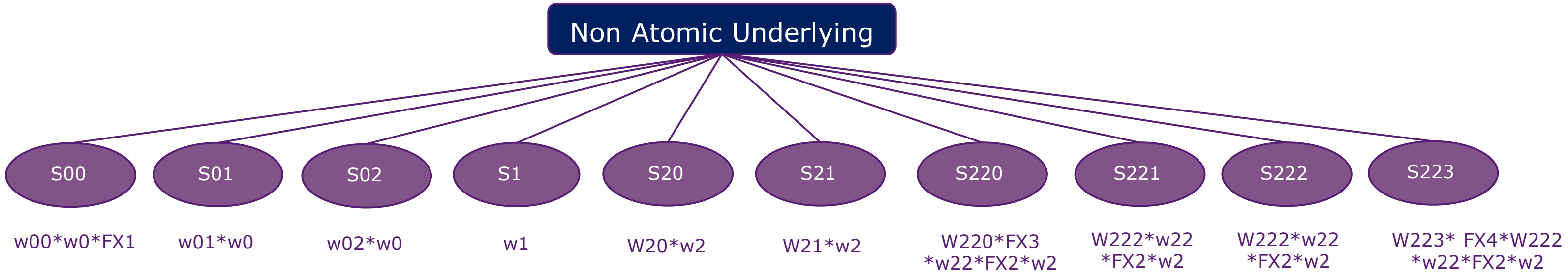
- **Shocks**
  Once the list of Obligors has been defined, we need to **assign a shock per Obligor.** In fact, the Obligors are not directly modelled within the pricing libraries, only the “non atomic” underlying is. Hence, we need to compute an equivalent shock : a shock of the “non atomic” underlying that would be observed if the Obligor were to default.

2. Look Through Approach

1) Build the tree



2) Flatten the tree



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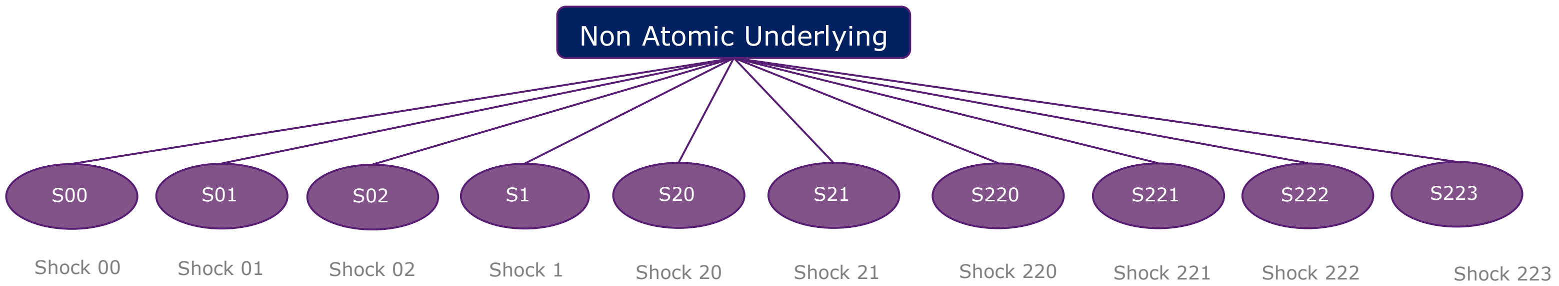
2. Look Through Approach

3) Calculate the shocks

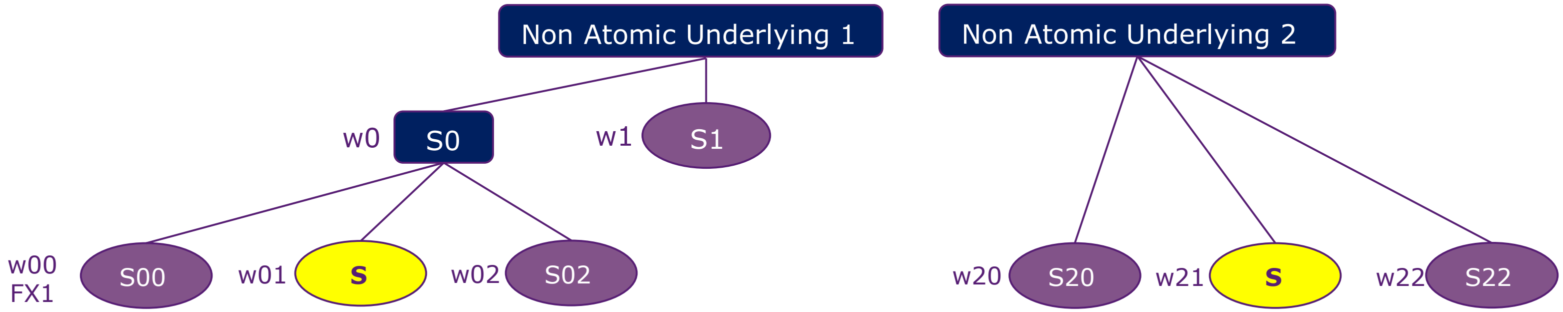
- Each shock is computed as the percentage with which the Non Atomic Underlying would vary if Obligor i were to default (hence Si= 0)



- Non equity underlyings are ignored (for instance, the Interest Rate Swaps within an auto-call) except for futures on dividends where the underlying are "indirectly" impacted by the default of the obligors

3. Additional Pricings

- Additional Pricings are required when one Obligor belongs to the composition of two or more underlyings of the instrument:



- Shocking each underlying separately to account for the Obligor default is economically not viable : this approach is incomplete due to the uncaptured cross-sensitivity.

A separate run where both underlyings are simultaneously shocked is required !

- This functionality has been implemented in the DRC algorithm. The latter captures the need for additional pricings, and proposes to the user to perform the computations.

- The Gross JTD may vary considerably depending on yes or no cross-sensitivities are taken into account.

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4. Monte Carlo Optimization

– Computations are performed **“within the pricing engines” and without additional simulations**. To illustrate the implemented methodology, let us consider a simple call option on a basket of n equities (S1,…, Sn) priced via MC (100K simulations) and on which we need to apply shocks (SH1, …, SHn) to calculate the Gross JTDs.

– The “**basic**” approach consists on:
  - looping over the shocks (SH1,…, SHn). For each shock:
    - -Shock the equity spot Si with the corresponding shock SHi at the data model level
    - -Create the pricer
    - -Simulate 100K trajectories
    - -Price the instrument

– The “**Advanced**” approach consists on:
  - building the pricer based on the baseline market context
  - simulating 100K trajectories
  - On each trajectory:
    - -Separately and consecutively apply the n shocks (SH1,…, SHn) then call the payoff
    - -Store the n intermediary calculations
  - Aggregate on the MC level for each shock

	Basic MC	Advanced MC
Pricer instances	n	1
Simulations	100K*n	100K
Payoff Calls	100K*n	100K*n

5. Pricing / Interpolation Grids

– Some non-atomic underlyings require too many repricings. Rather than performing all of them, the DRC algorithm performs maximum 11 repricings / underlying and interpolates the others.

– To illustrate this methodology, let us consider a call option on a basket of S&P 500 and FTSE 100 priced via MC (100K simulations). Calculating the Gross JTD for this option would on average require to apply 600 shocks.

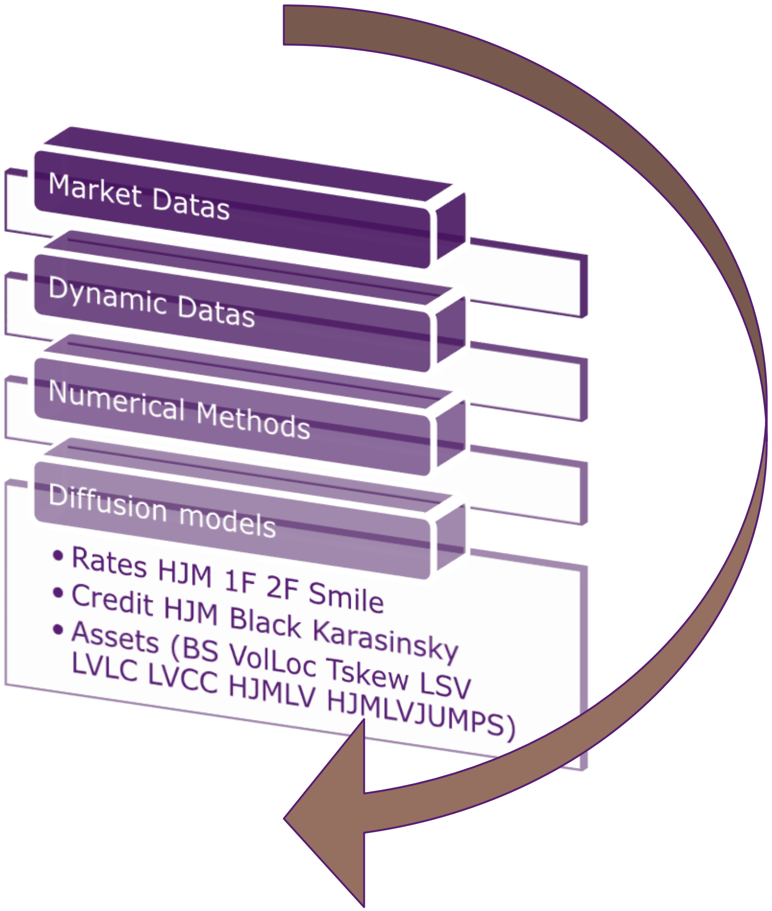
– Rather than performing all the shocks / pricings:

  - we identify the maximum and minimum shocks to apply to each underlying
  - set up a grid of 11 shocks for this underlying
  - perform the pricings using these grid shocks
  - Interpolate the prices corresponding to the non performed shocks (interpolation time is negligible)

In our example, we apply 22 shocks (rather than 600) then perform maximum 596 interpolations

	Basic MC without interpolation	Advanced MC with interpolation
Pricer instances	600	1
Simulations	600*100K	100K
Payoff	600*100K	22*100K

6. Change of the calling architecture



- From new market data context we jump directly to the trajectories
- We skip refining data (going from discrete points to continuous ones)
- We skip calibration of models
- We skip generation of random numbers
- We skip the path reconstruction
- All these steps make that the overhaul calculation is much more faster
- ……
- ??? Can we do it for non standard scenarios ???

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From one market data to another one

$$\frac{dS_t}{S_t} = \sigma(t, S_t)\,dW_t$$

$$\frac{d\widetilde{S_t}}{\widetilde{S_t}} = \tilde{\sigma}\left(t, \widetilde{S_t}\right)dW_t$$

$$\frac{d\widetilde{S_t}}{\widetilde{S_t}} = \frac{\tilde{\sigma}\left(t, \widetilde{S_t}\right)}{\sigma(t, S_t)}\frac{dS_t}{S_t}$$

- From new market data context we jump directly to the trajectories
- **No** : We skip refining data (going from discrete points to continuous ones)
- **No** : We skip calibration of models
- **Yes** : We skip generation of random numbers
- **Yes-simpler** : We skip the path reconstruction (simple replacement)
- All these steps make that the overhaul calculation is much more faster
- ……
- ??? Can we do it for non standard scenarios ???

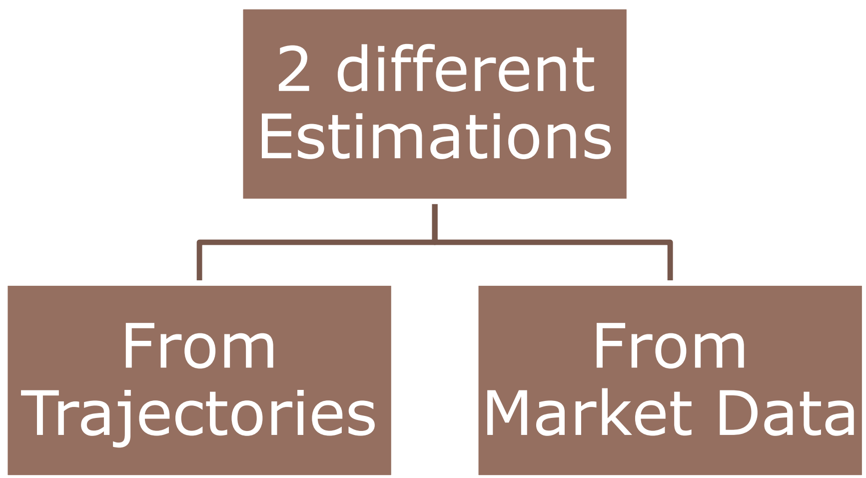
Approximate directly from Market data and Paths

$$\sigma(t, S_t) = \frac{\frac{\partial C}{\partial T}}{\frac{1}{2} K^2 \frac{\partial^2 C}{\partial K^2}} \approx \frac{\frac{\Sigma^2(T_i, K)T_i - \Sigma^2(T_{i-1}, K)T_{i-1}}{T_i - T_{i-1}}}{Digit(T, K)}$$

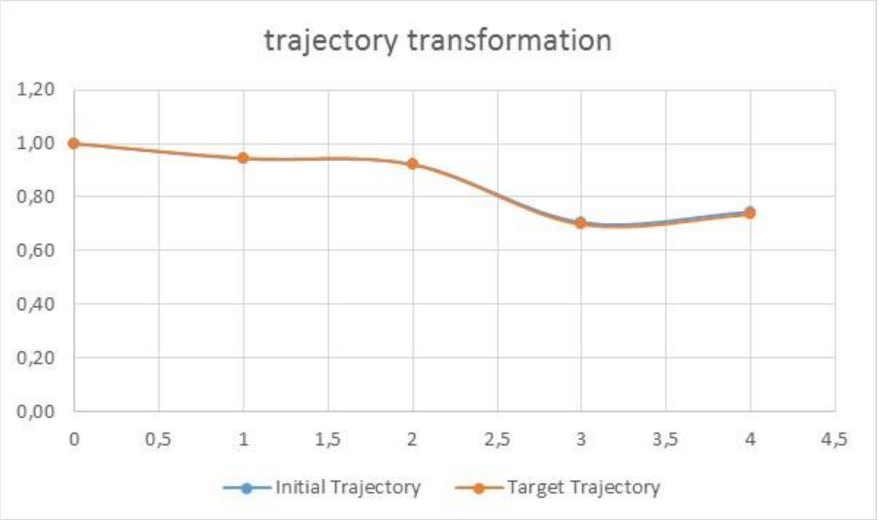
– From new market data context we jump directly to the trajectories

– We use this rule of thumb

➔ and transform existing trajectories into new ones without having to go through all library steps.



Example : Sanity check : go from 20% volatility 4000 samples to 20%



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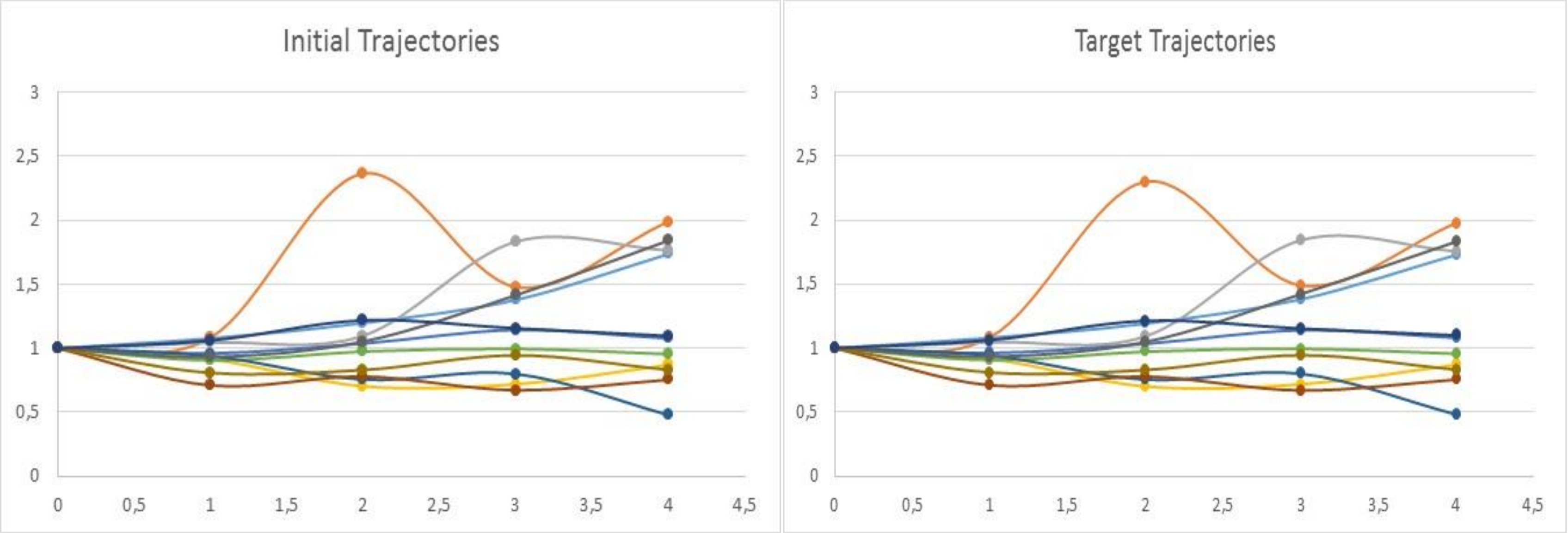
Example : Sanity check : go from 20% volatility 4000 samples to 20%



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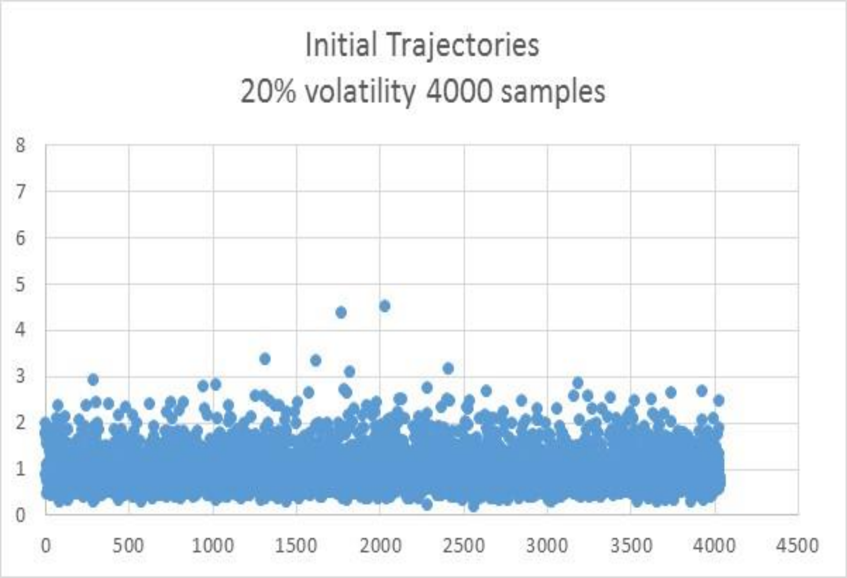

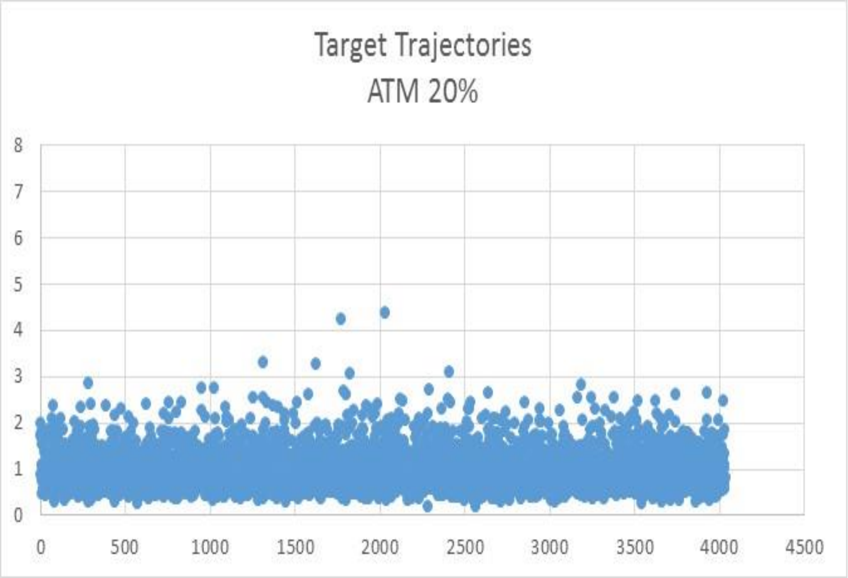

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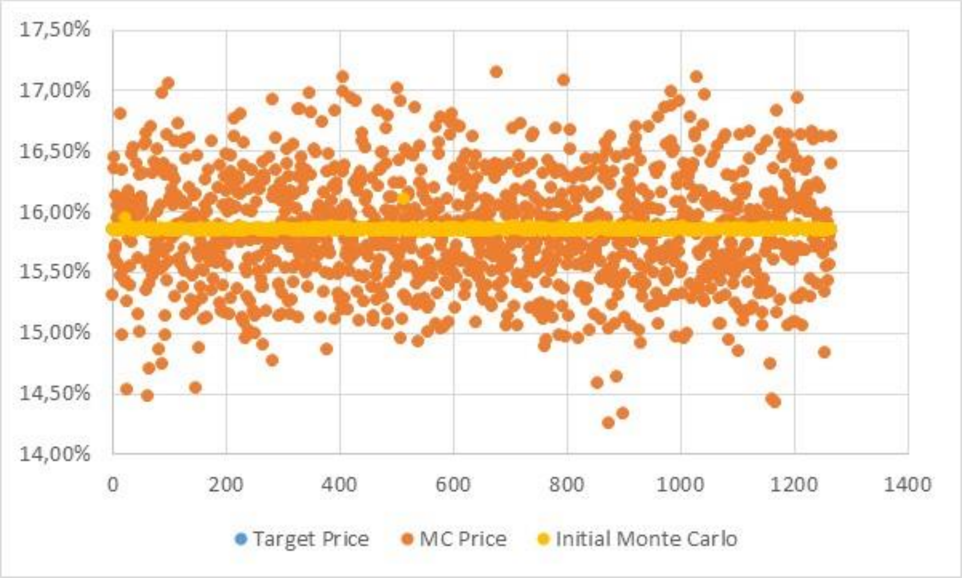
Example : Sanity check : go from 20% volatility 4000 samples to 20%

Strike	100%
Analytica Price	15,85%
Initial Monte Carlo	16,08%

Strikes	100%
IV	20,00%
Analytic Price	15,85%
Target Monte Carlo	15,86%

- Normal Monte Carlo with as little as 4000 samples does not converge to the basis point!
- The adjustment method seems in this example to erase this error.
- Can we repeat the experiment?

Example : Sanity check : repeat 1500 # Monte Carlo



- This method erases

Convergence error

Example : go from 30% volatility 4000 samples to 20% + Skew

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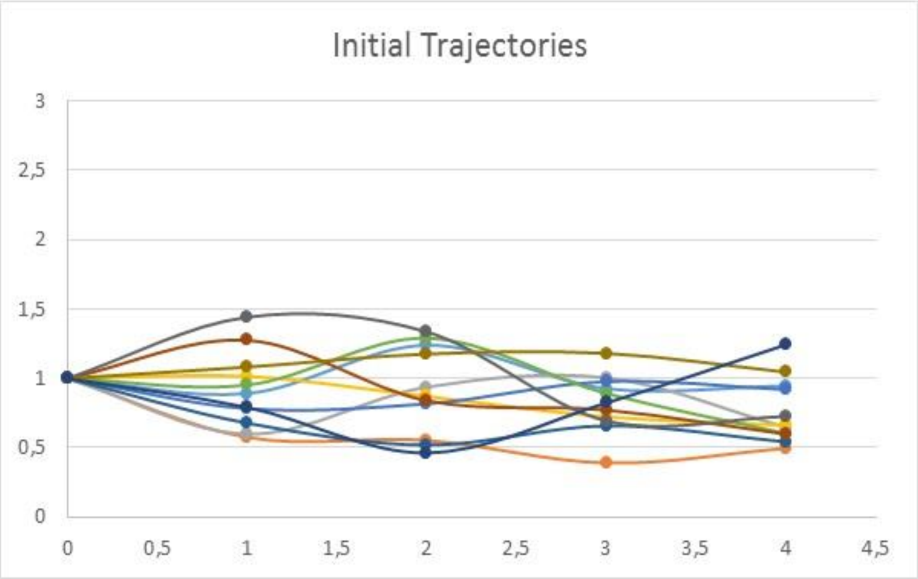
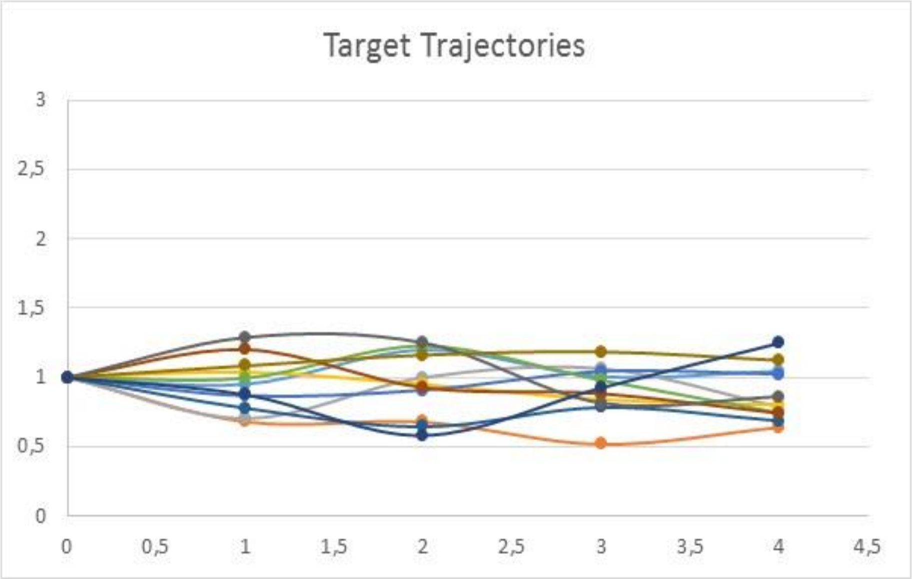
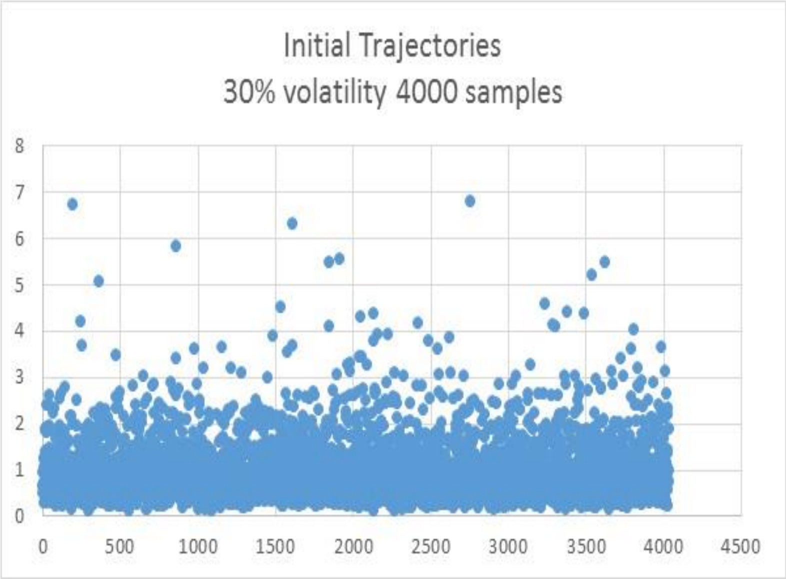
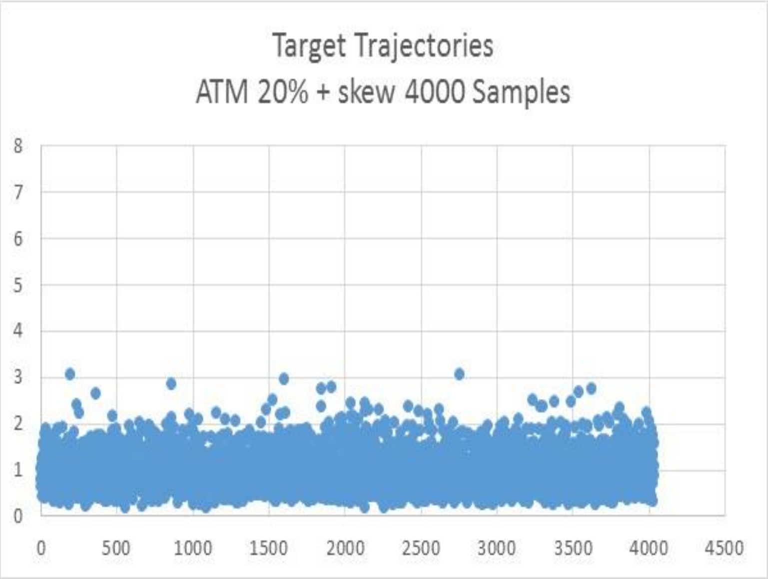
Example : go from 30% volatility 4000 samples to 20% + Skew





Example : go from 30% volatility 4000 samples to 20% + Skew





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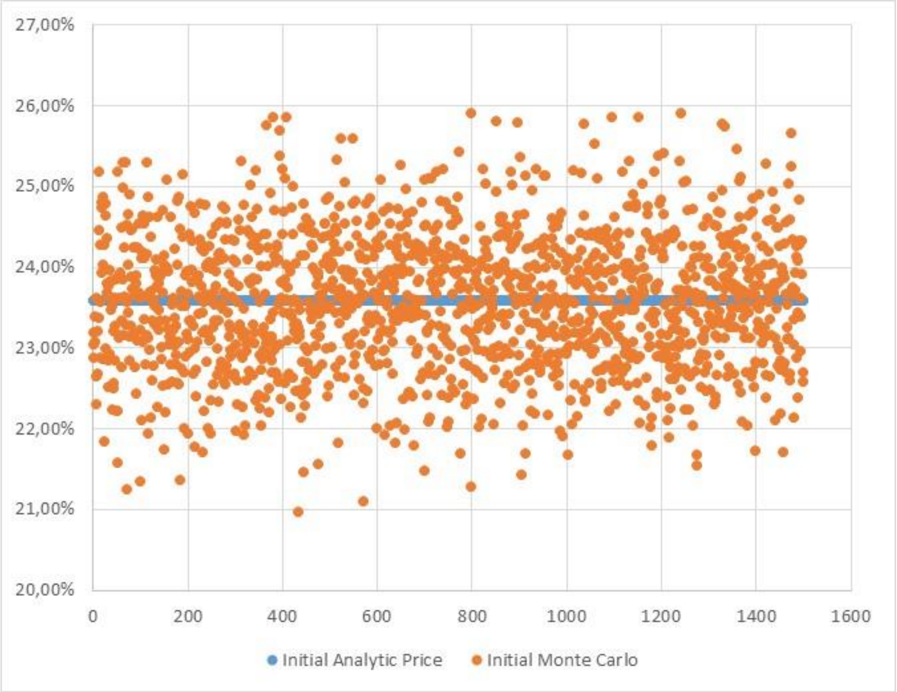
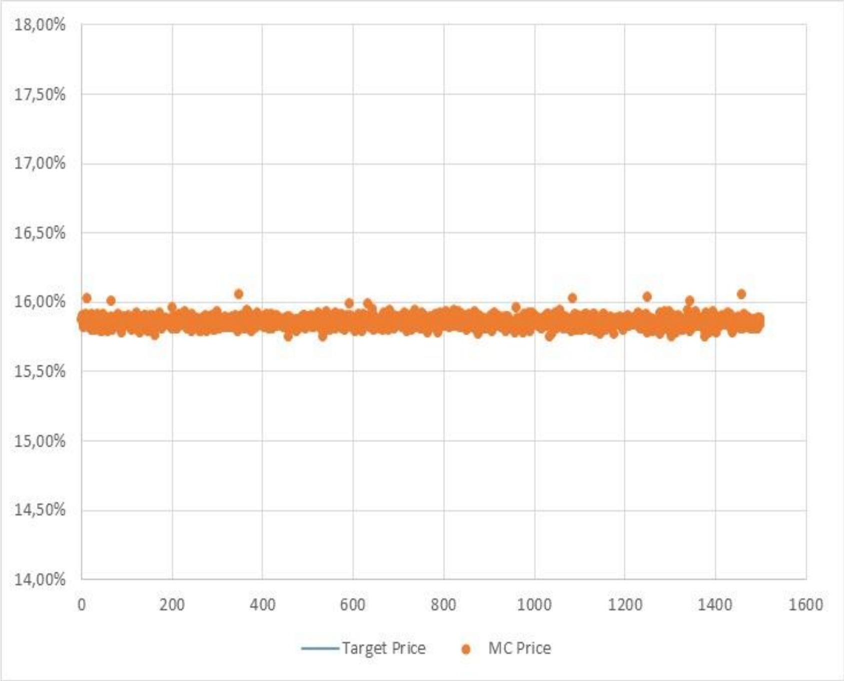
Example : go from 30% volatility 4000 samples to 20% + Skew

Strike	100%
Analytica Price	23,58%
Initial Monte Carlo	24,63%

Strikes	80%	100%	120%
IV	21,00%	20,00%	19,00%
Analytic Price	26,99%	15,85%	8,42%
Target Monte Carlo	27,00%	15,85%	8,42%

Example : repeat 1500 # Monte Carlo





- This method erases 3 types of Errors

- Calibration
- Discretisation
- Convergence

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Conclusion

- We have presented the computational challenge within the FRTB framework
- We have seen detailled examples solving the Standard method, precisely the DRC
- We have also presented several ideas to accelerate the pricing
  - Transforming the IT calling architecture
  - Hot Spot simulation & trajectory transformation

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