Robust (pointwise) theory of arbitrage pricing in discrete time

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based on a joint work with

M. Burzoni, M. Frittelli, Z. Hou and M. Maggis

Advances in Financial Mathematics, Paris, 10–13 Jan 2017

St John's College Oxford

Oxford Mathematics

OXFORD-MAN INSTITUTE

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erc European Research Council

Established by the European Commission

Outline

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Robust approach – an introduction

- Modelling in mathematical finance

Robust framework in discrete time

- General setup
- FTAP and related results
- Pricing-Hedging duality

American options

Robust approach to mathematical finance

UNIVERSAL MODEL ⟷ SPECIFIC MODEL

Universal Model

+ few assumptions, wide universe of scenarios
− non-unique outputs
+ sharpened by adding inputs

Specific Model

− strong assumptions, significant model risk
+ unique outputs
− often takes limited inputs

AIM: Develop a framework interpolating between the two and quantify the impact/risk of assumptions.

METHOD: Articulate assumptions as pathspace restrictions.

APPLICATIONS: Option pricing and hedging, optimal investment...

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An active field of research...

Explicit bounds on Price(G_T) and robust super-/sub- hedges in:
Hobson (98), Brown et al. (01), Dupire (05), Lee (07), Cox et al. (08), Cox and O. (11,11), Cox and Wang (12), Hobson and Klimmek (12,13), Galichon et al. (14), Cox, O. and Touzi (15), O. and Spoida (16), Henry-Labordère et al. (16)...

Arbitrage considerations and robust FTAP in:
Davis and Hobson (07), Cox and O. (11,11), Davis, O. and Raval (14), Acciaio et al. (16), Bouchard and Nutz (14), Burzoni, Frittelli and Maggis (15&16), Cheridito, Kupper, Tangpi (16), Burzoni, Frittelli, Hou, Maggis, O. (16) and ongoing...

Pricing-hedging duality in:
Davis, O. and Raval (14), Beiglböck, Henry-Labordère and Penkner (13), Neufeld and Nutz (13), Dolinsky and Soner (14), Tan and Touzi (13), Galichon et al. (14), Bouchard and Nutz (15), Possamaï et al. (14), Fahim and Huang (16), Bayraktar and Zhou (14), Cox, Hou and O. (16), Deng and Tan (16) and ongoing...

Pathspace restrictions $\mathfrak{P} \subsetneq \Omega$*:*

O. and Spoida (16), Hou and O. (16), Aksamit, Hou and O. (16), ...

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Robust setting in discrete time

Robust (pointwise/pathwise) setting:

- no frictions, all prices in discounted units, $\mathbb{T} := \{0, 1, ..., T\}$
- dynamically traded asset: $(S_t)_{t \in \mathbb{T}}$ valued in $\mathbb{R}^d$
- assets traded only at $t = 0$: $\Phi = \{\phi_1, \ldots, \phi_k\}$, $\mathbb{R}$-valued
- all assets defined on some Polish space X
  model = choice of scenarios $\Omega \subset X$

Robust interpolation of modelling choices:

- from *Universally acceptable* ($\Omega = X$)
  to $\mathbb{P}$– *Specific* ("$\Omega = \mathrm{supp}(\mathbb{P})$")
  $\rightsquigarrow$ rFTAP characterises "dead ends"
  i.e. Ωs incompatible with rational pricing theory,
  i.e. Ωs which do not support any martingale measure
- quantify outputs and their dependence on Ω
  $\rightsquigarrow$ requires pricing-hedging duality for any Ω

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Towards rFTAP...

A specification Ω of modelling beliefs is a "dead-end"
iff

$$\text{there are no calibrated martingale measures on } \Omega$$

iff

$$\text{any p-ty measure } \mathbb{P} \text{ on } \Omega \text{ admits a (classical) arbitrage}$$

This was called *weak arbitrage* by Davis and Hobson (07).

The first aim is to develop an intrinsic and non-circular understanding of such Ωs...

Arbitrage = feature of model specification $\neq$ trading strategy.
Arbitrage = all agents agree we can not price rationally (but may disagree on *why* it is so)

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Arbitrage Strategies

Given $\mathbb{F}$, $\mathcal{H}(\mathbb{F})$ are predictable processes w.r.t. $\mathbb{F}$ and

$$\mathcal{A}_\Phi(\mathbb{F}) = \left\{(\alpha, H) : \alpha \in L^0(\mathcal{F}_0, \mathbb{R}^k), H \in \mathcal{H}(\mathbb{F})\right\}$$

Trading in S: $(H \circ S)_T := \sum_{t=1}^T \sum_{j=1}^d H_t^j (S_t^j - S_{t-1}^j) = \sum_{t=1}^T H_t \cdot \Delta S_t$
Trading in Φ: $\alpha \cdot \Phi := \sum_{j=1}^k \alpha_j \phi_j$

Fix a filtration $\mathbb{F}$, $\Omega \in \mathcal{F}^{\mathcal{A}}$ and Φ.

1p A One-Point Arbitrage (1p-Arbitrage) is a strategy $(\alpha, H) \in \mathcal{A}_\Phi(\mathbb{F})$ such that $\alpha \cdot \Phi + (H \circ S)_T \geq 0$ on Ω with a strict inequality for some $\omega \in \Omega$.

SA A Strong Arbitrage is a strategy $(\alpha, H) \in \mathcal{A}_\Phi(\mathbb{F})$ such that $\alpha \cdot \Phi + (H \circ S)_T > 0$ on Ω.

$\mathbb{P}$A A (classical) $\mathbb{P}$-Arbitrage is a strategy $(\alpha, H) \in \mathcal{A}_\Phi(\mathbb{F}^{\mathbb{P}})$ such that $\alpha \cdot \Phi + (H \circ S)_T \geq 0$ $\mathbb{P}$-a.s. and > 0 with positive $\mathbb{P}$-ty.

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Martingale measures and efficient models

We consider the set of calibrated martingale measures:

$$\mathcal{M}_{\Omega,\Phi}(\mathbb{F}) := \{\mathbb{Q} \mid S \text{ is an } \mathbb{F}\text{-martingale under } \mathbb{Q}, \mathbb{Q}(\Omega) = 1, \mathbb{E}_{\mathbb{Q}}[\phi] = 0\ \forall \phi \in \Phi\}$$

Then

$$\mathcal{M}_{\Omega,\Phi}(\mathbb{F}) = \mathcal{M}_{\Omega,\Phi}(\mathbb{F}^S) \quad \forall \mathbb{F} : \mathbb{F}^S \subset \mathbb{F} \subset \mathbb{F}^M$$

where $\mathbb{F}^S$ is the natural filtration of S and

$$\mathbb{F}^M := (\mathcal{F}_t^M)_{t\in\mathbb{T}}, \quad \text{where } \mathcal{F}_t^M := \bigcap_{P\in\mathcal{M}_{\Omega,\Phi}(\mathbb{F}^S)} \mathcal{F}_t^S \vee \mathrm{NullSets}^P(\mathcal{F}_T^S).$$

We write $\mathcal{M}^f_{\Omega,\Phi}(\mathbb{F})$ for finitely supported measures. Let

$$\Omega^*_\Phi := \left\{\omega \in \Omega \mid \exists Q \in \mathcal{M}^f_{\Omega,\Phi} \text{ such that } Q(\omega) > 0\right\} = \bigcup_{Q\in\mathcal{M}^f_{\Omega,\Phi}} supp(Q)$$

so that $\Omega^*_\Phi \subset \Omega$ is an efficient model specification associated to Ω.

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Robust (pointwise) FTAP

Theorem

Fix $\Omega \in \mathcal{F}^{\mathcal{A}}$ and Φ a finite set of statically traded options. Then, there exists a filtration $\widetilde{\mathbb{F}}$ such that $\mathbb{F}^S \subseteq \widetilde{\mathbb{F}} \subseteq \mathbb{F}^M$ and

No Strong Arbitrage in $\mathcal{A}_\Phi(\widetilde{\mathbb{F}})$ on $\Omega \iff \mathcal{M}_{\Omega,\Phi} \neq \emptyset \iff \Omega^*_\Phi \neq \emptyset$.

Further, $\Omega^*_\Phi \in \mathcal{F}^{\mathcal{A}}$ and there exists an Arbitrage Aggregator $(\alpha^*, H^*) \in \mathcal{A}_\Phi(\widetilde{\mathbb{F}})$ such that $\alpha^* \cdot \Phi + (H^* \circ S)_T \geq 0$ on Ω and the following representation holds

$$\Omega^*_\Phi = \{\omega \in \Omega \mid \alpha^* \cdot \Phi(\omega) + (H^* \circ S)_T(\omega) = 0\}.$$

In fact, (α^*, H^*) and the resulting $\widetilde{\mathbb{F}}$ constructed explicitly (up to a measurable selection)!

Further, we have no 1p Arbitrages in $\mathcal{A}_\Phi(\mathbb{F}^{\mathcal{A}})$ iff $\Omega = \Omega^*_\Phi$.

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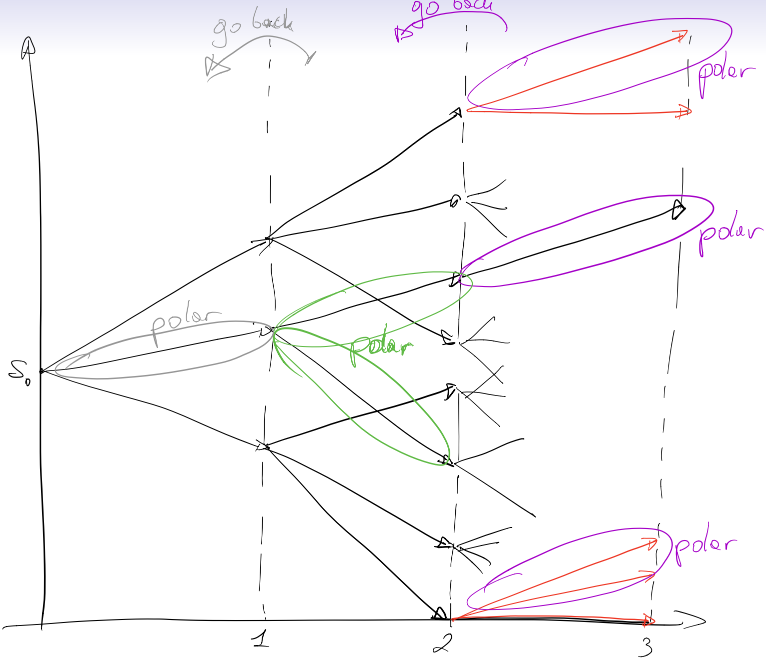
go back
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polar
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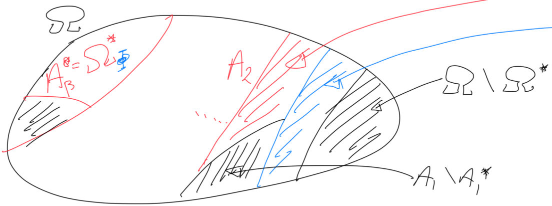
S_0
1
2
3

Pathspace Partition Scheme



Suppose there is a portfolio $(H^1 \cdot S)_T + \alpha^1 \cdot \Phi \geq 0$ on Ω^* with $= 0$ on A_1 & > 0 on $\Omega^* \setminus A_1$

Note that on A_1^*, $\alpha \cdot \Phi$ is replicable.

Say we have $(H^2 \cdot S)_T + \alpha^2 \cdot \Phi \geq 0$ on A_1^* with $= 0$ on A_2 & > 0 on $A_1^* \setminus A_2$

... & iterate $A_2 \rightsquigarrow A_2^* \rightsquigarrow A_3 \rightsquigarrow \dots \rightsquigarrow A_\beta \rightsquigarrow A_\beta^*$

each time we take α^i lin. independent so that a new $\alpha^i \cdot \Phi$ is redundant $\Rightarrow$ we stop after $\beta \leq k$ steps and either have arbitrage aggregator or $A_\beta^* \neq \emptyset$

PROP: $A_\beta^* = \Omega_\Phi$

Classical FTAP

We recover a version of the DMW theorem as a special case:

Proposition

Consider a probability measure $\mathbb{P}$ on X and let $\mathcal{M}^{\ll\mathbb{P}} := \{Q \in \mathcal{M} \mid Q \ll \mathbb{P}\}$. There exists a set of scenarios $\Omega^{\mathbb{P}} \in \mathcal{F}^{\mathcal{A}}$ and a filtration $\widetilde{\mathbb{F}}$ such that $\mathbb{F}^S \subseteq \widetilde{\mathbb{F}} \subseteq \mathbb{F}^M \subseteq \mathbb{F}^{\mathbb{P}}$ and

$$\text{No Strong Arbitrage in } \mathcal{A}(\widetilde{\mathbb{F}}) \text{ on } \Omega^{\mathbb{P}} \iff \mathcal{M}^{\ll\mathbb{P}} \neq \emptyset.$$

Further,

$$\text{No } \mathbb{P}\text{–arbitrage} \iff \mathbb{P}\left((\Omega^{\mathbb{P}})^*\right) = 1 \iff \mathcal{M}^{\sim\mathbb{P}} \neq \emptyset,$$

where $\mathcal{M}^{\sim\mathbb{P}} := \{Q \in \mathcal{M} \mid Q \sim \mathbb{P}\}$.

FTAP in natural filtration

"Compactness + continuity $\rightsquigarrow \widetilde{\mathbb{F}} = \mathbb{F}^S$"

We extend and generalise the results of Acciaio et al (13), see also Cheridito et al (16). Let $\Phi = \{\phi_i : i \in I\}$ with

$$\phi_i = g_i \circ S \quad \text{for some continuous } g_i : \mathbb{R}_+^{d\times(T+1)} \to \mathbb{R}, \quad \forall i \in I$$

and $\phi_0 = g_0(S_T)$ for a strictly convex super-linear function g_0 on $\mathbb{R}^d$, such that other options have a slower growth at infinity. Denote $\widetilde{\mathcal{M}}_{\Omega,\Phi} := \{Q \in \mathcal{M}_{\Omega,\Phi\setminus\{\phi_0\}} \mid E_Q[\phi_0] \le 0\}$.

Theorem

Consider $\Omega \in \mathcal{F}^{\mathcal{A}}$ such that $\Omega = \Omega^*$, $\pi_{\Omega^*}(\phi_0) > 0$ and $\exists\, \omega^* \in \Omega$ such that $S_0(\omega^*) = S_1(\omega^*) = \ldots = S_T(\omega^*)$. TFAE:

(1) *There is no Uniformly Strong Arbitrage on Ω in $\mathcal{A}_\Phi(\mathbb{F}^{\mathcal{A}})$;*

(2) *There is no Strong Arbitrage on Ω in $\mathcal{A}_\Phi(\mathbb{F}^{\mathcal{A}})$;*

(3) $\widetilde{\mathcal{M}}_{\Omega,\Phi} \neq \emptyset$.

Pricing–Hedging duality

For any set of scenarios $\Gamma \in \mathcal{F}^{\mathcal{A}}$, we define the superhedging price

$$\pi_{\Gamma,\Phi}(g) := \inf\left\{x \in \mathbb{R} \mid \exists(\alpha, H) \in \mathcal{A}_{\Phi}(\mathbb{F}^{A}) \text{ s.t. } x + \alpha \cdot \Phi + (H \circ S)_T \geq g \text{ on } \Gamma\right\}.$$

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$$\pi_{\Omega^*_{\Phi},\Phi}(g) = \sup_{\mathbb{Q} \in \mathcal{M}^f_{\Omega,\Phi}} \mathbb{E}_{\mathbb{Q}}[g]$$

and, if finite, the left hand side is attained by some strategy $(\alpha, H) \in \mathcal{A}_{\Phi}(\mathbb{F})$.

Note that in general $\pi_{\Omega,\Phi}(g) > \pi_{\Omega^*_{\Phi},\Phi}(g)$.

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Pricing and hedging of American options

(with Aksamit, Deng and Tan)

Consider now an American option ξ which pays ξ_t when exercised at time t. The hedger can switch strategy when the buyer exercises: let $\overline{\mathcal{H}}(\mathbb{F})$ denote $T-$tuples $H^{(1)}, \ldots, H^{(T)}$ of processes in $\mathcal{H}(\mathbb{F})$ s.t. $H_i^{(t)} = H_i^{(t+1)}$, $i \leq t \leq T$.

$$\pi^A_{\Omega^*,\Phi}(\xi) = \inf\{x : \exists \alpha, \overline{H} \in \overline{\mathcal{H}}(\mathbb{F}^A) \text{ s.t. } x + \alpha\cdot\Phi + (H\circ S)_T \geq \xi_t \text{ on } \Omega^*\}$$

We may have, cf. Hobson & Neuberger, Bayraktar & Zhou,

$$\pi^A_{\Omega^*,\Phi}(\xi) > \sup_{\mathbb{Q}\in\mathcal{M}_{\Omega,\Phi}} \sup_{\tau\in\mathcal{T}(\mathbb{F}^S)} \mathbb{E}_{\mathbb{Q}}[\xi_\tau],$$

where $\mathcal{T}(\mathbb{F})$ are $\mathbb{F}$–stopping times.

This is due to inability of models in $\mathcal{M}_{\Omega,\Phi}$ to adjust dynamically!

Proposition. Duality gap $\Longleftrightarrow$ lack of DPP for superhedging

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Step 1: turn Americans into Europeans

Following the insight of Deng and Tan, we proceed as follows:

- let $\overline{\Omega} := \Omega \times \mathbb{T}$, where $\mathbb{T} = \{1, 2, \ldots, T\}$. We write $\overline{\Omega} \ni \overline{\omega} = (\omega, \theta)$ and let $\mathtt{t}(\overline{\omega}) = \theta$
- Let $\overline{\mathcal{F}}_j = \mathcal{F}^S_j \vee \sigma(\{\mathtt{t} \le i\} : i \le j) = \mathcal{F}^{\overline{S}}_j$ for $\overline{S}_t = (S_t, \mathbf{1}_{\mathtt{t} \le t})$
- Observe that $\overline{\Omega}^* = \Omega^* \times \mathbb{T}$ and $\overline{\mathcal{H}}(\mathbb{F}^S) = \mathcal{H}(\overline{\mathbb{F}})$
- Define $\overline{\xi}$ on $\overline{\Omega}$ by $\overline{\xi}(\overline{\omega}) = \xi(\omega)_\theta$ we have

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Step 1: turn Americans into Europeans

Following the insight of Deng and Tan, we proceed as follows:

- let $\overline{\Omega} := \Omega \times \mathbb{T}$, where $\mathbb{T} = \{1, 2, \ldots, T\}$. We write $\overline{\Omega} \ni \overline{\omega} = (\omega, \theta)$ and let $\mathtt{t}(\overline{\omega}) = \theta$
- Let $\overline{\mathcal{F}}_j = \mathcal{F}^S_j \vee \sigma(\{\mathtt{t} \leq i\} : i \leq j) = \mathcal{F}^{\overline{S}}_j$ for $\overline{S}_t = (S_t, \mathbf{1}_{\mathtt{t} \leq t})$
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In Summary...

- We propose an abstract modelling framework based on pointwise/pathwise specification of scenarios
- This ranges from Model-Independent to Model-Specific
- We establish a robust FTAP which captures both extremes
- We establish Pricing-Hedging duality for measurable claims
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Thank You

Preprint available at: arXiv:1612.07618

DAVIS & HOBSON '07 EXAMPLE REVISITED

$X = \mathbb{R}_+^T$ & $S_t(\omega) = \omega_t$ is the canonical process. $\Omega = X$

$S_0(\omega) = S_0$

$\Phi = \{(S_T - K_1)^+ - c,\ (S_T - K_2)^+ - c\}$ for some $0 < K_1 < K_2$ and $c > 0$

$\Longrightarrow \forall\ \mathbb{P} \in \mathcal{M}_\Phi$ $\mathbb{E}^{\mathbb{P}}[(S_T - K_1)^+] = \mathbb{E}^{\mathbb{P}}[(S_T - K_2)^+] = c > 0$

$\Longrightarrow \mathcal{M}_\Phi$ is empty

$\Omega \rightsquigarrow \Omega^* = \Omega \longmapsto$ Observe that $\underbrace{(1,-1)}_{\alpha^1} \cdot \Phi \geq 0$ on Ω with

$= 0$ on $A_1 = \{\omega : S_T \leq K_1\}$

> 0 otherwise

$\rightsquigarrow K_1 \leq S_0 \Longrightarrow A_1^* \leq A_1$ or

$K_1 > S_0 \Longrightarrow A_1^* = A_1$ $\rightsquigarrow \underbrace{(0,-1)}_{\alpha^2} \cdot \Phi \geq 0$ on A_1

$\Longrightarrow A_2 = \emptyset$

$\Longrightarrow$ Arbitrage aggregator: $\alpha^1 \cdot \Phi + \alpha^2 \cdot \Phi \cdot \mathbb{1}_{A_1}$